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T and S dualities and the cosmological evolution of the dilaton and the scale factors

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Abstract. Cosmologically stabilizing radion along with the dilaton is one of the major concerns of low energy string theory. One can hope that T and S dualities can provide a plausible answer. In this work we study the impact of S and T duality invariances on dilaton gravity. We have shown various instances where physically interesting models arise as a result of imposing the mentioned invariances. In particular S duality has a very privileged effect, in that the dilaton equations partially decouple from the evolution of the scale factors. This makes it easy to understand the general rules for the stabilization of the dilaton. We also show that certain T duality invariant actions become S duality invariance compatible. That is, they mimic S duality when the extra dimensions stabilize.

1 Introduction

Extra dimensions and the dilaton are an integral part of string theory. But in a cosmological setting one has to have them stabilized in order to recover Einstein gravity at low energies. Stabilization of extra dimensions and the dilaton has been the focus of extensive study in the literature [1–32]. But there is, as yet, no general consensus on the mechanism of their stabilization nor on whether a single mechanism should be responsible for the stabilization of both. T and S dualities are important symmetries of string theory. In this paper we study their effect on a generalized dilaton gravity action.

Stabilizing the radion in pure Einstein gravity is not difficult. But Einstein theory is not the proper low energy limit of string theory. In reality one has to deal with the dilaton and this problem proves to be rather nontrivial. Recently an explicit model has been presented [33] to achieve this in a way that fits in string theory. The observation there was that since T duality plays a crucial role in the stabilization of the radion, S duality might as well provide an understanding of the stabilization of the dilaton. In the mentioned paper it was shown that this could be the case. Here we would like to use the same approach but generalize the model to an arbitrary dilaton theory for which the contributions to the matter lagrangian, for instance the effect of D-branes, are all of perfect fluid form.

To contrast our main point clearer on the evolution of the dilaton and its stabilization, we shortly review a recent model [24–26] of the cosmology of D-branes and the dilaton. The reason we are doing this is because *dilaton stabilization* and that *a low energy theory is compatible with a constant dilaton* are rather different things. We will see that such models are a viable possibility. The action of the model in the mentioned works is

$$S=\int \mathrm{d}x^d\sqrt{-g}\mathrm{e}^{-2\phi}[R+4(\nabla\phi)^2+\mathrm{e}^{a\phi}\mathcal{L}]\,. \tag{1}$$

We take our metric to be (assuming all directions are flat)

$$\mathrm{d}s^2=-\mathrm{d}t^2+\mathrm{e}^{2B(t)}\sum_{b=1}^{m}\mathrm{d}x_b^2+\mathrm{e}^{2C(t)}\sum_{c=1}^{p}\mathrm{d}y_c^2\,, \tag{2}$$

where B and C denote the cosmological scale factors of the m-dimensional observed and p-dimensional extra dimensional spaces, respectively. In a model motivated by string theory one has $m+p=9$ and $d=1+m+p=10$.

The term $\mathcal{L}$ in the action (1) refers to the contribution of D-branes, and a describes their coupling to the dilaton. In the dilute gas approximation (for details see for instance [21, 22]) the D-brane action has the form of a hydrodynamical fluid,

$$\mathcal{L}=-2\sum_i\rho_i\,, \tag{3}$$

where the energy densities are given as follows

$$\rho_i=\rho_i^0\exp[-(1+\omega_i)mB-(1+\nu_i)pC]\,, \tag{4}$$

with ω_i and ν_i representing the pressure coefficients of the corresponding contribution ρ_i.

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The equations of motion following from this Lagrangian will be

$$\ddot{B}+k\dot{B}=\mathrm{e}^{a\phi}[T_{\hat{b}\hat{b}}-\tau\rho]\,, \tag{5a}$$
$$\ddot{C}+k\dot{C}=\mathrm{e}^{a\phi}[T_{\hat{c}\hat{c}}-\tau\rho]\,, \tag{5b}$$
$$\ddot{\phi}+k\dot{\phi}=\frac{1}{2}\mathrm{e}^{a\phi}[T-(d-2)\tau\rho]\,, \tag{5c}$$
$$k^2=m\dot{B}^2+p\dot{C}^2+2\mathrm{e}^{a\phi}\rho\,, \tag{5d}$$
$$k\equiv m\dot{B}+p\dot{C}-2\dot{\phi}\,, \tag{5e}$$

Here $T_{\hat{b}\hat{b}}$ and $T_{\hat{c}\hat{c}}$ represent, in the orthonormal frame, the total pressure in the m-dimensional observed and p-dimensional compact spaces, respectively. If the sources have the fluid form they are given by

$$T_{\hat{b}\hat{b}}=\sum_i \omega_i\rho_i\,, \tag{6a}$$
$$T_{\hat{c}\hat{c}}=\sum_i \nu_i\rho_i\,. \tag{6b}$$

T represents the trace of the total energy momentum tensor and it is given by

$$T=\sum_i(-1+m\omega_i+p\nu_i)\rho_i\,. \tag{7}$$

We have also used $\tau\equiv(a-2)/2$ for compact notation.

The model considers two contributions to the total energy density:

$$\text{winding}\qquad \rho_{\mathrm{w}}=\rho_{\mathrm{w}}^0\mathrm{e}^{-mB}, \tag{8a}$$
$$\text{momentum}\qquad \rho_{\mathrm{m}}=\rho_{\mathrm{m}}^0\mathrm{e}^{-mB-(1+p)C} \tag{8b}$$

corresponding to the following pressure coefficients (see for instance [21, 22] for their derivation),

$$\text{winding}\qquad \omega=0\qquad \nu=-1\,,$$
$$\text{momentum}\qquad \omega=0\qquad \nu=1/p\,.$$

The equations admit the following solution for $m=3$ and $a=1$ (the dilaton coupling to D-branes from string theory):

$$C=C_0\,, \tag{9a}$$
$$\phi=\phi_0\,, \tag{9b}$$
$$B=\frac{2}{3}\ln(t)+B_0\,, \tag{9c}$$

with

$$\mathrm{e}^{-(1+p)C_0}=\frac{\rho_{\mathrm{w}}^0}{\rho_{\mathrm{m}}^0}\left(\frac{p}{p+2}\right), \tag{10a}$$
$$\mathrm{e}^{\phi_0-3B_0}=\frac{2}{3\rho_{\mathrm{w}}^0}\left(\frac{p+2}{p+1}\right). \tag{10b}$$

We would like to point out the following points for our subsequent discussion.

- The D-brane action in (1) is not T nor S duality invariant (to be explicitly defined in the next chapter) with the matter terms given in (8). So T and S duality invariances are explicitly broken by this model of D-branes.
- A constant dilaton is admissible only for $m=3$ and this happens because for $m=3$ and $a=1$ the right-hand side of the equation with ϕ becomes proportional to the right-hand side of the equation with C. Hence the stabilization of the extra dimensions makes the right-hand side of the equations with ϕ identically zero. So it is the equation with C that triggers this. We will understand this effect in terms of S duality in the next section.
- ϕ_0 and B_0 are not separately fixed. Only the combination ϕ_0-3B_0 is fixed by the constraint (zero–zero component of the tensor equation).

The solution is stable in the sense that if one perturbs the solution presented above, the variations can be found as follows:

$$\delta C=\frac{\alpha}{\sqrt{t}}\sin[\Omega\ln(t)+\beta]\,, \tag{11a}$$
$$\delta B=-\frac{\gamma}{6}-\frac{\beta}{3t}\ln(t)\,, \tag{11b}$$
$$\delta\phi=\frac{p\alpha}{2\sqrt{t}}\sin[\Omega\ln(t)+\beta]+\frac{\beta}{2t}-\frac{\gamma}{2}\,, \tag{11c}$$

with α, β and γ small arbitrary constants and $\Omega=\sqrt{(4p+5)/3}/2$.

So no perturbation grows in time. Furthermore the constant arbitrary shifts on B and ϕ are compatible with the initial constraint. That is,

$$\delta\phi-3\delta B\to 0\quad\text{as}\quad t\to\infty\,,$$

which shows that the initial constraints for stabilization will not be altered. Therefore, this model is fully compatible with a fixed dilaton. How this happens can be considered to be an accident, but $m=3$ and $a=1$ are not arbitrary fine tunings of the parameters.

One would like to have a fixed dilaton for late time cosmologies in order to recover Einstein gravity. As recently argued [33] with an explicit example S duality can actually help fix the dilaton (determine ϕ_0), because it is a symmetry that directly acts on it. But in the mentioned work the stabilization of extra dimensions is achieved via the same element as the one that stabilizes the dilaton. As this short review shows, there can be models which are compatible with a fixed dilaton and yet are solely responsible for the stabilization of extra dimensions. Even though Occam's razor would direct us to a more economical explanation, we should nevertheless keep in mind that stabilization of extra dimensions and the dilaton can be achieved via different mechanisms.

2 General formalism

In what follows we will use a similar action

$$S=\int \mathrm{d}x^{10}\sqrt{-g}\mathrm{e}^{-2\phi}[R+4(\nabla\phi)^2+\mathcal{L}]\,. \tag{12}$$

But for the interaction we will take the following generalization:

$$\mathcal{L} = -2\sum_i \mathrm{e}^{a_i\phi} V_i(\phi)\rho_i\,, \tag{13}$$

with ρ_i again having the fluid form.[1] The generalization compared to the example in the first chapter is that now we allow for a general coupling of the dilaton to various matter contributions, not just D-branes. Also, as is evident from our definition of the dilaton coupling to different ρ, the $V_i(\phi)$ should not be homogeneous in e^{ϕ}. This will avoid any ambiguity in the definition of the a_i.

The equations of motion following from (12) and (13) are

$$\ddot{B} + k\dot{B} = \mathcal{B} - \mathcal{V}'\,, \tag{14a}$$
$$\ddot{C} + k\dot{C} = \mathcal{C} - \mathcal{V}'\,, \tag{14b}$$
$$\ddot{\phi} + k\dot{\phi} = \mathcal{F} - \frac{d-2}{2}\mathcal{V}'\,, \tag{14c}$$
$$k^2 = m\dot{B}^2 + p\dot{C}^2 + 2\mathcal{E}\,, \tag{14d}$$
$$k \equiv m\dot{B} + p\dot{C} - 2\dot{\phi}\,, \tag{14e}$$

with the following definitions (where we take $\tau_i = (a_i-2)/2$)

$$\mathcal{E} = \sum_i \mathrm{e}^{a_i\phi} V_i(\phi)\rho_i\,, \tag{15a}$$
$$\mathcal{V}' = \frac{1}{2}\sum_i \mathrm{e}^{a_i\phi}\frac{\mathrm{d}V_i}{\mathrm{d}\phi}\rho_i\,, \tag{15b}$$
$$\mathcal{B} = \sum_i \mathrm{e}^{a_i\phi} V_i\left[T^i_{\hat{b}\hat{b}} - \tau_i\rho_i\right]\,, \tag{15c}$$
$$\mathcal{C} = \sum_i \mathrm{e}^{a_i\phi} V_i\left[T^i_{\hat{c}\hat{c}} - \tau_i\rho_i\right]\,, \tag{15d}$$
$$\mathcal{F} = \frac{1}{2}\sum_i \mathrm{e}^{a_i\phi} V_i\left[T^i - (d-2)\tau_i\rho_i\right]\,. \tag{15e}$$

Even though we implicitly started out with flat observed and internal dimensions, the effect of curvature in these spaces can easily be accommodated within the formalism of the fluid approach. Quite generally we can simulate the effect of a curvature term via the following energy density:

$$\rho_K = -Kq\exp[-(1+\omega_K)mB - (1+\nu_K)pC]\,, \tag{16}$$

where $q = \{m, p\}$ is the dimensionality of the partition where there is curvature $K = \{<0, 0, >0\}$ representing hyperbolic, flat and spherical geometries, respectively. The pressure coefficient of the partition having curvature will be $(2-q)/q$ and the other one -1. For the curvature terms we of course have $a_K = 0$. However one should not forget that this is just a sleight of hand. The curvature terms should be understood as such only at the level of the equations of motion, not the action level, since in reality they are part of R (the marble) and not a part of some energy density in the interaction Lagrangian (the wood).

Furthermore, a pure dilaton potential has $\omega_p = \nu_p = -1$ with an arbitrary potential $V_p(\phi)$ and a_p.

3 Impact of T and S dualities

3.1 S duality

The kinetic term in the action is invariant under S duality, which is given by

$$\phi \to -\phi\,, \tag{17a}$$
$$g_{\mu\nu} \to \mathrm{e}^{-\phi} g_{\mu\nu}\,. \tag{17b}$$

In terms of our metric ansatz this means that

$$\phi \to -\phi\,, \tag{18a}$$
$$B \to B - \phi/2\,, \tag{18b}$$
$$C \to C - \phi/2\,. \tag{18c}$$

The remaining part is not generally S invariant, so we will have to impose conditions. As a remark we would like to point out that S duality invariance cannot be imposed on the interactions as an interplay between two terms: this would necessarily imply the equality of the respective pressure coefficients $\omega_1 = \omega_2$ and $\nu_1 = \nu_2$, meaning that the two terms can be combined with a new potential $V = V_1 + V_2$. The effect of S duality invariance on each term gives the following condition:

$$V_i(-\phi) = V_i(\phi)\,, \tag{19a}$$
$$\tau_i = \frac{1}{8}[-1 + m\omega_i + p\nu_i]\,. \tag{19b}$$

This implies the following. S duality invariance means $\mathcal{F} = 0$. Thus we have a partial decoupling of the dilaton. This means that the conditions for ϕ stabilization can be orthogonal to the conditions for C stabilization.

We now have a better understanding of why the model presented in the introduction is compatible with a fixed dilaton: it *mimics* S duality invariance when C is stabilized.

3.1.1 Stabilization

We can now look for general conditions for the stabilization of extra dimensions and the dilaton. We must have the following:

$$\mathcal{V}'(\phi_0, C_0) = 0\,, \tag{20a}$$
$$\mathcal{C}(\phi_0, C_0) = 0\,. \tag{20b}$$

A decoupling of the conditions will arise when $\mathcal{V}'(\phi_0, C_0) = 0$ is achieved for

$$\left.\frac{\mathrm{d}V_i}{\mathrm{d}\phi}\right|_{\phi_0} = 0\,. \tag{21}$$

[1] It would appear that we took $H_{\mu\nu\lambda} = 0$. But we assume that their effect can also be summarized as a hydrodynamical fluid.

To study the stability of this fixed point we can perturb the solutions and look how they evolve. The equation for the perturbations will be (we assumed a simple power law for the unperturbed solution for B)

$$\delta\ddot{X}+\frac{\alpha}{t}\delta\dot{X}=-\frac{1}{t^2}\Sigma\delta X\,. \tag{22}$$

Here α is a model dependent number and δX is a column vector with $\delta X^{\mathrm{T}}=(\delta B,\delta C,\delta\phi)$. We call Σ the stabilization matrix

$$\begin{pmatrix}\Sigma_{BB} & \Sigma_{BC} & \Sigma_{B\phi}\\ \Sigma_{CB} & \Sigma_{CC} & \Sigma_{C\phi}\\ \Sigma_{\phi B} & \Sigma_{\phi C} & \Sigma_{\phi\phi}\end{pmatrix}.$$

The elements are in essence the derivatives of the right-hand sides of the evolution equations evaluated at the stabilization point. For example, $\Sigma_{B\phi}$ is the first derivative with respect to ϕ of the right-hand side of the evolution equation for B evaluated at ϕ_0 and C_0 defined in (20a). All eigenvalues of Σ must be positive for the equilibrium point to be stable.

We must point out here that in order not to also have a static B forced on us by the conditions, one must either have all the ω_i equal to each other, or, to at least have stabilization at late times, there must be two ρ_i with the same a_i and V_i for the smallest ω in the model.[2] These two will in general be the momentum and winding modes of the object excited around extra dimensions. If these are met, B will expand presumably according to a power law solution depending on the smallest ω term. One can show that with this rather orthodox assumption the stabilization matrix simplifies to

$$\begin{pmatrix}\Sigma_{BB} & \Sigma_{BC} & \Sigma_{B\phi}\\ 0 & \Sigma_{CC} & \Sigma_{C\phi}\\ 0 & 0 & \Sigma_{\phi\phi}\end{pmatrix}.$$

The eigenvalues therefore are given by Σ_{BB}, Σ_{CC} and $\Sigma_{\phi\phi}$ and they all have to be positive. The $\phi\phi$ part is relatively easy, since the natural condition there just becomes

$$\left.\frac{\mathrm{d}^2V_i}{\mathrm{d}\phi^2}\right|_{\phi_0}>0\,. \tag{23}$$

The rest becomes a game related to whether stabilization of the extra dimensions can be achieved. However, we would like to point out that the tridiagonal form of Σ will prevent the mixing of δC and δB to $\delta\phi$, even if there is an instability in the evolution of the observed and extra dimensions. This comes as a special case circumventing the concerns presented in [29].

It is also rather nice to have dilaton stabilization via functions of the form $\mathrm{e}^{a_i\phi}V_i(\phi)\rho_i$. The reason is that if B is increasing (expanding observed universe) ρ_i will decrease and it will act as a factor that damps the magnitude of the potentials. This damping effect in concert with the ordinary friction term k already present in the equations of motion (Hubble damping) will help reduce violent oscillations of the evolutions of the scale factors and the dilaton.

In [33] a model which encompasses these general features have been presented. The protagonists for stabilization were (m,n) strings with an action covariant in S duality. By this we mean that the action consisted of the winding and the momentum modes with the momentum parts manifestly S duality invariant; however, the winding modes were not manifestly invariant by the transformation and an extra interchange of the quantum numbers $(m,n)\to(n,m)$ is required. This results in a spontaneous breaking of the S symmetry and ϕ_0 was fixed to be $\ln(n/m)$.

3.2 T duality

T duality acts on the compact dimensions and the dilaton. For simplicity let us assume that we act with it on the whole p-dimensional compact space. This means that we have

$$C\to -C\,, \tag{24a}$$
$$\phi\to\phi-pC\,. \tag{24b}$$

This leaves the kinetic part invariant. A term in the interaction part will be T dual if

$$a_i=2(1+\nu_i)\,, \tag{25a}$$
$$V_i(\phi)=1\,. \tag{25b}$$

This implies, as expected, that such a term will not contribute to the right-hand side of the equations with C.

Imposing further the S duality invariance condition in (19) we get the TS self-duality condition,

$$\omega_i=\frac{1+(m-1)\nu_i}{m}\,. \tag{26}$$

There are various cases, for example $\nu_i=0$ meaning that $\omega_i=1/m$ and $a_i=2$: some sort of fundamental string that behaves like radiation along the observed dimensions and pressureless dust along the extra dimensions. For $\nu_i=-1$ we get $\omega_i=(2-m)/m$ and $a_i=0$. This case is analogous to a curvature term along the observed dimensions except that we must remember that a *real* curvature term will have $\rho_K^0=-mK$. Thus such a term can have interplay with a real curvature term.

3.2.1 T duality invariance as interplay between two terms

One could argue that, unlike S duality, imposing T duality on a single term is not a must, since its essence is about the contrast between winding and momentum modes. If one imposes T duality as an interplay between two terms one gets

$$\omega_1=\omega_2\,, \tag{27a}$$
$$a_1=a_2=2+(\nu_1+\nu_2)\,, \tag{27b}$$
$$V_2=V_1=1\,, \tag{27c}$$
$$\rho_1^0=\rho_2^0\,. \tag{27d}$$

[2] The smallest w terms will be redshift slowest and will become the dominant terms at late times in cosmology.

An interesting case is $\nu_2 = -\nu_1$, giving $a_i = 2$: winding and momentum modes of a fundamental string. This results in the following equations:

$$\ddot{B} + k\dot{B} = 2\omega f(\phi, B, C)\cosh(\nu pC)\,, \tag{28a}$$

$$\ddot{C} + k\dot{C} = -2\nu f(\phi, B, C)\sinh(\nu pC)\,, \tag{28b}$$

$$\ddot{\phi} + k\dot{\phi} = (m\omega - 1) f(\phi, B, C)\cosh(\nu pC) - p\nu f(\phi, B, C)\sinh(\nu pC)\,, \tag{28c}$$

with $f(\phi, B, C) = \rho^0 \mathrm{e}^{2\phi-(1+\omega)mB-pC}$. As is evident the radion will stabilize at $C = 0$ and this gets rid of one term in the ϕ equations. The system will be compatible with a fixed dilaton if $\omega = 1/m$, which is like radiation along the observed dimensions. This example is like the one presented in the introduction: C stabilization *mimics* S duality invariance for a specific albeit physically meaningful choice of parameters.

4 A note on the cosmological constant and acceleration

A pure cosmological constant has $a_\Lambda = 0$ and $\omega_\Lambda = \nu_\Lambda = -1$.[3] Therefore, this term is T invariant but not S invariant. The equations read

$$\ddot{B} + k\dot{B} = 0\,, \tag{29a}$$

$$\ddot{C} + k\dot{C} = 0\,, \tag{29b}$$

$$\ddot{\phi} + k\dot{\phi} = -\Lambda\,, \tag{29c}$$

$$k^2 = m\dot{B}^2 + p\dot{C}^2 + 2\Lambda\,. \tag{29d}$$

The solutions are $B = B_0$, $C = C_0$ and $\phi = \pm\sqrt{\Lambda/2}t$.[4] Therefore, the string coupling $g_\mathrm{s} = \mathrm{e}^\phi$ is accelerating and the spatial dimensions are static. However, this is the picture in the string frame. When one converts to an Einstein frame one recovers only $B_\mathrm{E} = \ln(t_\mathrm{E}) + B_{0\mathrm{E}}$ and $C_\mathrm{E} = \ln(t_\mathrm{E}) + C_{0\mathrm{E}}$. These are not accelerating. This curious result shows that it is not always possible to get acceleration with a cosmological constant.

One can think of an S invariant pure dilaton potential term without a constant accompanying V. This will have $\tau_{\Lambda_\phi} = -5/4$ and $a_{\Lambda_\phi} = -1/2$ along with $\omega_{\Lambda_\phi} = \nu_{\Lambda_\phi} = -1$ and $V_{\Lambda_\phi} = \Lambda_\phi$. The equations read in this case

$$\ddot{B} + k\dot{B} = \frac{1}{4}\Lambda_\phi \mathrm{e}^{-\phi/2}, \tag{30a}$$

$$\ddot{C} + k\dot{C} = \frac{1}{4}\Lambda_\phi \mathrm{e}^{-\phi/2}, \tag{30b}$$

$$\ddot{\phi} + k\dot{\phi} = 0\,, \tag{30c}$$

$$k^2 = m\dot{B}^2 + p\dot{C}^2 + 2\Lambda_\phi \mathrm{e}^{-\phi/2}. \tag{30d}$$

These are compatible with a constant dilaton and hence the Einstein frame and the string frame become identical. Therefore, $\Lambda_\phi \mathrm{e}^{-\phi_0/2}$ acts like an effective cosmological constant in Einstein gravity and we recover accelerating solutions. It is interesting to speculate on a possible interplay between Λ_ϕ and Λ, the former being only S invariant and the latter being only T invariant. In these cases one recovers the following equations:

$$\ddot{B} + k\dot{B} = \frac{1}{4}\Lambda_\phi \mathrm{e}^{-\phi/2}\,, \tag{31a}$$

$$\ddot{C} + k\dot{C} = \frac{1}{4}\Lambda_\phi \mathrm{e}^{-\phi/2}\,, \tag{31b}$$

$$\ddot{\phi} + k\dot{\phi} = -\Lambda\,, \tag{31c}$$

$$k^2 = m\dot{B}^2 + p\dot{C}^2 + 2\Lambda_\phi \mathrm{e}^{-\phi/2} + 2\Lambda\,. \tag{31d}$$

Of course, in this scenario one cannot stabilize the dilaton. However, if initially we are in a regime where the dilaton is decreasing as $\phi = -\sqrt{\Lambda/2}t$ (similar to the first example in this section) the scale factors will evolve as if they are under the influence of an ever increasing cosmological constant. This is an interesting example in that it could require much less time to inflate to the required number of e-foldings. A saner scenario will be a non-constant dilaton S invariant potential $V(\phi)$ along with a cosmological constant;

$$\ddot{B} + k\dot{B} = \frac{1}{4}\mathrm{e}^{-\phi/2}V - \frac{1}{2}\mathrm{e}^{-\phi/2}V'\,, \tag{32a}$$

$$\ddot{C} + k\dot{C} = \frac{1}{4}\mathrm{e}^{-\phi/2}V - \frac{1}{2}\mathrm{e}^{-\phi/2}V'\,, \tag{32b}$$

$$\ddot{\phi} + k\dot{\phi} = -\Lambda - 2\mathrm{e}^{-\phi/2}V'\,, \tag{32c}$$

$$k^2 = m\dot{B}^2 + p\dot{C}^2 + 2\mathrm{e}^{-\phi/2}V + 2\Lambda\,. \tag{32d}$$

The dilaton acts as if it is under the influence of an effective potential,

$$\frac{\mathrm{d}V_\mathrm{eff}}{\mathrm{d}\phi} = \Lambda + 2\mathrm{e}^{-\phi/2}\frac{\mathrm{d}V}{\mathrm{d}\phi}\,. \tag{33}$$

It is apparent that since Λ explicitly breaks S invariance it is not possible to stabilize the dilaton at $\phi_0 = 0$ even if V does not spontaneously break S symmetry. If the dilaton can be stabilized the scale factors will evolve as if they are under the influence of an effective cosmological constant $\Lambda_\mathrm{eff} = \Lambda + \mathrm{e}^{-\phi_0/2}V(\phi_0)$. But we emphasize again that this is due to the presence of V_ϕ; if it is absent we will be back to the case at the beginning of this chapter and one will not have acceleration.

5 Conclusion

The key observation we have presented is that *T duality invariant actions can for certain physically relevant cases mimic S duality invariance when internal dimensions stabilize* and thus the resulting model will be equivalent to Einstein gravity since one has a constant dilaton (unspecified value) and stabilized radion.

[3] The reason for choosing $a_\Lambda = 0$ for a cosmological constant is that Λ must come as an addition to R in the action.

[4] In string theory Λ (or c in [2]) can be negative. In that case the solutions are different.

Another important result that follows from our discussions and that justifies and strengthens a known fact is that it is in general wrong to assume that the dilaton can be taken to be a constant a priori. We have shown various examples of this, and possibly the most striking one is the impossibility of a constant dilaton for a cosmological constant. The dilaton can be stabilized with a potential and if the potential has a non-zero value at its minimum one recovers the ordinary interpretation of vacuum energy as cosmological constant.

We have made use of the most general possible interaction terms which are in the form of hydrodynamical fluid. We have seen that S duality invariance has to be imposed term by term in the interaction. Consequently the dilaton equation is partially decoupled from the other equations and this is of great help in studying radion and dilaton stabilization. T duality invariance on the other hand can be imposed term by term or as an interplay between two terms in the Lagrangian. If a single term in the interaction is T duality invariant it will not contribute to the evolution of the extra dimensions and hence is compatible with a constant radion albeit of unspecified value. If the sum of two terms is invariant under the T symmetry radion will evolve and will be stabilized at $C = 0$. We have shown some physically interesting cases of this sort that also trigger *mimicking* of S duality, and the stabilization of the radion field results in a theory compatible with a constant dilaton; hence one recovers Einstein's theory. It is also possible to have the opposite form: an S invariant action stabilizing the dilaton – that is fix the value at $\phi_0 = 0$ (or $\phi_0 \neq 0$ if there is room for spontaneous breaking of S symmetry) – and this action being compatible with a constant albeit unspecified value of the radion.

To conclude it is of crucial importance to know what mechanism is actually responsible for the stabilization of the dilaton and also whether this mechanism also triggers the stabilization of the dilaton or vice-versa. We see that T, S and TS invariant interactions provide a good starting point to resolve this issue.

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